Reg. No. : ....

Name : ....

K20P 1187

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.)
Examination, October 2020
(2017 Admission Onwards)
MATHEMATICS
MAT3C12 : Functional Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Show that K^n with the norm $\| \ \|_2$ is strictly convex.

2. For normed spaces X and Y, prove that $\|F\| = \sup \{\|F(x)\| : x \in X, \|x\| \leq 1\}$ is a norm on BL (X, Y).

3. Define continuous seminorm on a Banach space.

4. For a finite dimensional subspace Y of a normed space X, prove that there is a continuous projection P defined on X such that R(P) = Y.

5. For an inner product space X, prove parallelogram law.

6. Give an orthonormal basis for the inner product space l^2. **(4×4=16)**

PART – B

Answer **four** questions from this Part without omitting **any Unit**. **Each** question carries **16** marks.

UNIT – I

7. a) Prove that every closed and bounded subset of a normed space X is compact if and only if X is finite dimensional.

   b) If E_1 is open in a normed space X and $E_2 \subset X$ then show that $E_1 + E_2$ is open in X.

P.T.O.

8. a) State and prove Hahn Banach separation theorem.

   b) For a normed space X and a subspace Y of X, prove that $x \in \overline{Y}$ if and only if $x \in X$ and $f(x) = 0$ whenever $f \in X'$ and $f/Y = 0$.

9. a) Show that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

   b) Define Schauder basis for a normed space X. Also prove that if X has a schauder basis $\{x_1, x_2, ...\}$ then X must be separable.

UNIT – II

10. a) State and prove uniform boundedness principle.

    b) State and prove Resonance theorem.

11. a) For Banach spaces X and Y and a closed linear map $F : X \to Y$, show that F is continuous.

    b) If X is a normed space and P is a projection on X then prove that P is a closed map if and only if R(P) and Z(P) are closed in X.

12. a) State and prove bounded inverse theorem.

    b) Let X be a Banach space in the norm $\| \;\; \|$. Then prove that a norm $\| \;\; \|'$ on the linear space X is equivalent to the norm $\| \;\; \|$ if and only if X is also a Banach space in the norm $\| \;\; \|'$ and the norm $\| \;\; \|'$ is comparable to the norm $\| \;\; \|$.

UNIT – III

13. a) For an inner product space X, prove that for all $x, y \in X$, $|< x, y >|^2 \leq < x, x >< y, y >$.

    b) Explain Gram-Schmidt orthonormalization for an inner product space X.

14. a) Let E be a non-empty closed convex subset of a Hilbert space H. Then prove that for each $x \in H$, there exists a unique best approximation from E to x.

    b) Let X be an inner product space and $E \subset X$ is convex then prove that there exists at most one best approximation from E to any $x \in X$.

15. a) State and prove projection theorem.

    b) Show that projection theorem does not hold for the inner product space c_{00}. **(4×16=64)**