K25U 2350

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2025
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
5B06MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. State distributive property of multiplication over addition.
2. Define the absolute value of a real number a.
3. Give an example of a bounded monotone sequence.
4. Write the sequence of partial sums of the series $\sum \frac{1}{n}$.
5. Give an example of a function discontinuous only at $x = -3$.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, show that $a = 0$.
7. Express $\frac{1}{7}$ and $\frac{3}{7}$ as periodic decimals.
8. If $a > b$ and $b > c$, use order properties to show that $a > c$.

9. Show that a sequence in $\mathbb{R}$ can have at most one limit.

10. Give a formula for the n^{th} term of the sequences $x_n = (5, 7, 9, 11, ...)$ and $y_n = (6, 8, 10, 12, ...)$.

11. Find $\lim_{n \to \infty} \left(\frac{3n+1}{2n+3} \right)$.

12. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges.

13. Show that the series $\sum \cos n$ is divergent.

14. Show that an absolutely convergent series in $\mathbb{R}$ is convergent.

15. Given that $f, g : A \to \mathbb{R}$ are continuous on a set $A \subseteq \mathbb{R}$. Show that fg is continuous on A.

16. Given that $f(x) = \frac{x^2 - x - 6}{x - 3}$ for $x \neq 3$. Define f at $x = 3$ in such a way that f is continuous at $x = 3$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. State and prove Archimedean Property of the Set $\mathbb{N}$ of natural numbers.

18. Show that every contractive sequence is convergent.

19. Given that (x_n) and (y_n) are sequences of real numbers such that $(x_n) \to x$ and $(y_n) \to y$, where $x, y \in \mathbb{R}$. Show that $(x_n + y_n) \to x + y$ and $(x_n \cdot y_n)$ $x \cdot y$.

20. Define a Cauchy sequence. Show that a Cauchy sequence is convergent.

21. Establish the convergence or the divergence of the series whose n^{th} term is $\frac{n!}{3.5.7....(2n+1)}$.

22. Given that (z_n) be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Show that the alternating series $\sum(-1)^{n+1}z_n$ is convergent.

23. State and prove Preservation of Intervals Theorem.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. Show that there exists a positive real number x such that $x^2 = 2$.

25. State and prove Monotone Subsequence Theorem.

26. State and prove ratio test for convergence of a series $\sum x_n$.

27. Given that $I = [a, b]$ is a closed bounded interval and let $f : I \to \mathbb{R}$ is continuous on I. Show that f has an absolute maximum and an absolute minimum on I.