K21U 3610

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS – Supple.) Examination, April 2021
(2017 – 2018 Admission)
CORE COURSE IN MATHEMATICS
2B02MAT – Integral Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. State mean value theorem for definite integrals.
2. Evaluate $\int_2^3 x^2 dx$.
3. Define gamma integral.
4. What is Fubini's theorem (First form) ?

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Evaluate $\sum_{k=1}^{4} k^2 - 3k$.
6. Show that Beta function is symmetric.
7. Find the average value of $f(x) = 4 - x^2$ on [0, 3].
8. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
9. Prove that $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\Gamma(n)}{a^n} (a > 0 ; n > 0)$.
10. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$.
11. Find $\frac{d^2y}{dx^2}$ if $x = t - t^2$ and $y = t - t^3$.

P.T.O.

12. Evaluate $\int_0^1 \frac{2}{\sqrt{3+4x^2}}\,dx$.

13. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)dy\,dx$.

14. Find the length of the cardioid $r = 1 - \cos\theta$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Use the inequality $\cos x \geq 1 - \frac{x^2}{2}$, which holds for all x, to find a lower bound for the value of $\int_0^1 \cos x\,dx$.

16. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

17. Find the area of the region in first quadrant that is bounded above by $y = \sqrt{x}$, and below by the x – axis and the line $y = x - 2$.

18. Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.

19. Find the average value of $f(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant.

20. Find the tangent to the right-hand hyperbola branch $x = \sec t$, $y = \tan t$, $\frac{-\pi}{2} < t < \frac{\pi}{2}$ at the point $\left(\sqrt{2}, 1\right)$ where $t = \frac{\pi}{4}$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Show that $\beta(m, n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.

22. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

23. Find the centroid ($\delta = 1$) of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy plane.

24. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.