K22U 0127

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Supple./Improv.) Examination, April 2022
(2016 – 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B10MAT – Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. Define subspace of a vector space.
2. Is $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 1, y)$ a linear transformation ?
3. Define column nullity of a matrix.
4. Solve the system of linear equations.

$$2x - 3y + z = -1$$
$$-3y - z = -9$$
$$5z = 15$$

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. Show that zero vector in a vector space is unique.
6. Define linearly dependent set. Show that $S = \{(1, 0, 2), (0, 1, -1), (2, 0, 0)\}$ linearly independent set in $\mathbb{R}^3$.
7. Define null space and range of a linear transformation.
8. Let V and W be vector spaces over the field F and let $T, U : V \to W$ be linear. For all $a \in F$, show that $aT + U$ is linear.

P.T.O.

9. Solve the system of equations

   $x - y + z = 0$

   $x + 2y - z = 0$

   $2x + y + 3z = 0.$

10. Show that the equations

    $2x + 6y = -11$

    $6x + 20y - 6z = -3$

    $6y - 8z = -1$ are not consistent.

11. Show that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

12. Show that the characteristic roots of a Hermitian matrix are all real.

13. Solve the system

    $2x + y + z = 10$

    $3x + 2y + 3z = 18$

    $x + 4y + 9z = 16$ by the Gauss-Jordan method.

14. Show that the characteristic polynomial of any diagonalizable linear operator T splits.

SECTION – C

Answer **any four** questions, **each** question carries 4 marks.

15. Let W be a subspace of finite dimensional vector space V. Prove that W is finite dimensional and $\dim(W) \leq \dim(V)$.

16. Let V be a vector space and S be a subset generates V. If β is a maximal linearly independent subset of S. Prove that β is a basis for V.

17. Let V and W be vector spaces of equal finite dimension and let $T : V \to W$ be linear. Prove that T is one-to-one if and only if T is onto.

18. Find the characteristic roots of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

19. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$.

20. Show that $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ is diagonalizable and find the diagonal form.

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

21. State and prove Replacement theorem for a basis of a vector space.

22. Let $U : P_3(\mathbb{R}) \to P_2(\mathbb{R})$ by $U(f) = f'$ and $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ by $T(f) = \int_0^x f\,dx$ be linear transformations. Let $\alpha = \{1, x, x^2, x^3\}$ and $\beta = \{1, x, x^2\}$ be basis of $P_3(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. Show that $[UT]_\beta = [U]_\alpha^\beta [T]_\beta^\alpha = [I]_\beta$.

23. Investigate for what values of λ, μ the simultaneous equations :

$x + 2y + z = 8$

$2x + y + 3z = 13$

$3x + 4y - \lambda z = \mu$

have a) no solution b) a unique solution and c) infinitely many solutions.

24. Using modified Gauss method, find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.
