K16U 0621

Reg. No.: ....

Name: ....

IV Semester B.Sc. Degree (CBCSS – 2014 Admn. Regular) Examination, May 2016
Core Course in Mathematics
4B04 MAT : ELEMENTS OF MATHEMATICS – II

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. How many relations are there on the set {a, b, c} ?
2. Give an example of a finite partially ordered set having neither a first element nor a last element.
3. What do you mean by conjugate points with respect to a conic ?
4. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. **(4×1 = 4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Give a relation which is neither symmetric nor antisymmetric and another one that is both symmetric and antisymmetric.
6. Let A = {1, 2, 3, ..., 15}. Let R be the equivalence relation on A defined by congruence modulo 4. Find the equivalence classes determined by R.
7. Let f, g : Z → Z be functions defined by f(x) = 2x + 3 and g(x) = 3x + 2. Obtain f $\circ$ g and g $\circ$ f.

8. Show by example that a set which is not totally ordered may contain a linearly ordered subset.

9. Let A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by the relation "x divides y". Draw the Hasse diagram of A.

10. Find the coordinates of the point of intersection of tangents drawn to $y^2 = 4ax$ at the points where it is cut by the straight line $x \cos\alpha + y \sin\alpha = p$.

11. Find the condition for the lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ to be conjugate with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

12. Show that the product of the perpendiculars from any point of a hyperbola to its asymptotes is constant.

13. Find the value of x such that the matrix $A = \begin{bmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{bmatrix}$ has rank 2.

14. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form. **(8×2 = 16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each.**

15. Let $R = \{(a, b) \in P \times P : a \geq b \text{ and } a \leq 3\}$ where P is the set of positive integers. Is it a partial order ? Justify your answer.

16. Let $f : R^+ \to [-5, \infty)$ be defined by $f(x) = 9x^2 + 6x - 5$ where R^+ is the set of positive real numbers. Find a formula for f^{-1}. Determine $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(y)$.

17. Let D = {1, 2, 3, 5, 6, 10, 15, 30} be ordered by divisibility.

   a) Which elements are join-irreducible ?

   b) Which elements are atoms ?

   c) Find a complement of 5, if it exists.

   d) Express 30 as the join of a minimum number of irredundant join-irreducible elements.

18. Find the equation of the pair of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$.

19. Chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$. Find the locus of their poles.

20. Using elementary row transformations, compute the inverse of the matrix

$$A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ 4 & 1 & 8\end{bmatrix}.$$

(4×4 = 16)

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Let A be a set of nonzero integers and let $\approx$ be the relation on A defined by $(a, b) \approx (c, d)$ whenever $ad = bc$. Determine whether $\approx$ is an equivalence relation on A. If it is so, find the equivalence class of (3, 2).

22. Let L be a finite distributive lattice. Show that every $\alpha \in L$ can be written uniquely, except for order, as the join of irredundant join-irreducible elements.

23. Show that the locus of the mid-points of normal chords of the parabola $y^2 = 4ax$ is $\frac{y^2}{2a}+\frac{4a^3}{y^2}=x-2a$.

24. For the matrix $A=\begin{bmatrix}1 & -1 & 2 & -1\\ 4 & 2 & -1 & 2\\ 2 & 2 & -2 & 0\end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in the normal form.

(2×6 = 12)